A Local Structure Graph Model: Formation of Network Edges as a Function of Other Edges

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Abstract

Localized network processes are central to the study of political science, whether in the the formation of political coalitions and voting blocks, balancing and bandwagoning, policy learning, imitation, diffusion, tipping-point dynamics, or cascade effects. These types of processes are not easily modeled using traditional network approaches, which focus on *global* rather than *local* structures within networks. We show that localized network processes, in which network edges form in response to the formation or characteristics of other edges, are best modeled by reconceptualizing edges (e.g., an alliance) as network nodes, and relationships among edges (e.g., belonging to the same neighborhood) as edges among these nodes. We propose a theoretical framework for modeling these processes and a statistical estimator that corresponds to this framework—a local structure graph model (LSGM). We demonstrate the properties of LSGMs using Monte Carlo simulations and explore action–reaction processes in two empirical applications: formation of alliances among countries and legislative cosponsorships in the US Senate.

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1 Introduction

Social actors rarely act independent of other's influences. Legislators confer before important votes (Kingdon 1973; Matthews and Stimson 1975) and seek one another's cosponsorships on legislation (Kirkland and Gross 2014); international firms form networks of production locations in many different countries (Echandi, Krajcovicova, and Qiang 2015); countries are embedded within dense networks of trade, inter-governmental organizations (IGOs), and alliances (Maoz 2006, 2010; Chyzh 2016; Hays, Schilling, and Boehmke 2015). Growing theoretical attention to the study of interdependence has, in turn, created a demand for more appropriate methodological tools that directly model such interdependence—a demand that spurred burgeoning statistical research in network analysis (Franzese, Hays, and Kachi 2012; Gile and Handcock 2015; Minhas, Hoff, and Ward 2016).

This paper builds on this research by focusing on modeling of what we refer to as *localized network processes*—a class of theoretical processes involving the formation of network structures as a function of other structures that form within the network. If we think of actors as nodes within the network, and pairwise relationships among them as edges, then in the simplest form localized network processes would include the formation of network edges as a function of other edges within the network. The location affects these types of action–reaction processes: *where* within the network an action occurs affects the likelihood of reaction to it. Localized network processes include the formation of coalitions and voting blocs, balancing and bandwagoning, policy learning, imitation, diffusion, tipping-point dynamics, and cascade effects.

Central theories behind coalition formation, for example, emphasize that alignments within coalitions (edges) happen in response to formation of alignments (edges) within a rival coalition. Voting blocs in legislatures form to balance opposing blocs or bandwagon as one coalition rushes to provides support to their co-partisans. Thus, if we think of legislators as nodes within a network, and an act of cosponsoring piece of legislation as a formation of an edge with the sponsor and other cosponsors, then the relational processes of interest may involve bandwagoning—additional cosponsorships by other ideationally similar legislators—or balancing—an introduction of a competing bill cosponsored by legislators from the opposite part of the ideological spectrum. Analogously, alliances among countries frequently form with the goal of balancing against an alliance of a political rival: e.g., the Soviet bloc formed the Warsaw Pact in response to the US and its allies forming the North Atlantic Treaty Organization (NATO) in the aftermath of World War II.

Baccini and Dür (2012) posit a similar theoretical process behind the formation of preferential trade agreements, arguing that pairs of countries sign mainly as response to the preferential trade agreements formed by other countries, with which they are competing for exports. Political parties frequently form coalitions, or join together to share manifestos and ballot lines. Lobby groups have been shown to form in response to other special-interests forming lobbies of their own (Gray and Lowery 2001).

Such substantively important processes, however, are not easily modeled using the traditional network approaches, such as exponential random graph models (ERGMs) or latent space models. ERGMs are focused on *global* or system-level rather than *localized* network configurations. The resulting inferences related to the global structure of the network (i.e, the probability of occurrence of particular structures within the network) do not allow for *local*-level insights (i.e., *where* in the network are these structures most likely to be observed) (Casleton, Nordman, and Kaiser 2016). Latent space models, which aim at accounting for unobserved unit- and edge-level effects, are not easily adaptable for modeling localized processes.

The theoretical framework proposed here relies on relaxing the pre-determined assumptions of what constitutes a node and/or an edge within a network. We argue that identification of nodes and edges must be tailored to the specific empirical application. Thus, while in traditional approaches actors are treated as network nodes and relationships among them as edges, modeling localized network processes may require, for example, treating edges (e.g., an alliance) as network nodes, and connectivities among edges (e.g., belonging to the same neighborhood) as (second-degree) edges. The proposed theoretical framework emphasizes connectivities among edges, or second-degree connectivities, in a general sense. The source of connectivity may stem from discrete edge characteristics (e.g., two edges are connected if they share a common node or if they connect two similar nodes) or may be measured on a continuous scale (e.g., intensity/strength of connections among edges depends on a continuous dyad-of-edges-level attribute).

We demonstrate that the theoretical processes of interest—processes that involve realizations of network edges as conditional on realizations of other edges—may be estimated using an adaptation of a class of network/spatial models known in statistics as local structure graph models (LSGMs), that fall within the broader class of Markov random field models (MRFs) (Casleton, Nordman, and Kaiser 2016). To formulate an LSGM, one must first specify a set of full conditional distributions for each potential edge in the network, i.e. the distribution of the presence/absence of an edge given the outcomes for all potential edges and a set of exogenous covariates. Thus, each conditional distribution is specified in terms of a neighborhood structure that explicitly identifies the degree of "local" dependency between all pairs of edges within the network. As we show in the article, a set of conditional distributions can determine a joint probability model for the network, under some rather general conditions. We use Monte Carlo simulations to demonstrate that the model allows for practical estimation of parameters that are easily interpretable. Finally, we supplement simulations with two empirical applications: the formation of the alliance network among countries between 1946–2007 and the formation of the network of legislative cosponsorships on labor-related legislation in the Senate of the 107^{th} US Congress.

2 Current Approaches to Modeling Interdependence

Network analysis has found wide application in all sub-fields of political science. Scholars have applied network tools to derive insights on the functioning of legislatures (Cho and Fowler 2010), diffusion of policies (Desmarais, Harden, and Boehmke 2015), and international conflict and cooperation (Kinne 2013). Network analysis has informed both theory-building and inference. Thus, some research employs network game theory and agent-based modeling with the goal of tracing the process of network formation and studying the properties of networks of interest (Jackson 2008; Chyzh 2016; Gallop 2016; Maoz and Joyce 2016; Siegel 2009). Other research focuses on development and application of network-informed probabilistic estimators that would allow for deriving statistical inferences. Such probabilistic network modeling may be further classified into exponential random graph models (ERGMs), latent space models (LSMs), and spatial or conditional autoregressions (SARs and CARs).

ERGMs account for network dependencies via the inclusion of covariates that play the role of sufficient statistics and correspond to specific *global* topological features of the graph (e.g., reciprocity, triads, 2-stars). These network topologies, also know as Markovian features, are defined as counts of all elements of a certain class weighed as a proportion of the total count that could potentially form in the given graph. The parameters associated with such covariates will then inform us of the prevalence of each type of element in the observed realization of the graph (Wasserman and Faust 1994; Carrington, Scott, and Wasserman 2005).

In contrast to our approach, ERGMs provide the most leverage for modeling global network dependencies rather than those localized to specific parts of the network. While some theoretical processes are easily modeled via global or network-level Markovian features, the localized network processes such as coalition-building, balancing and bandwagoning, or spatial diffusion are less amenable to the ERGMs framework. Whereas ERGMS assume homogeneous effects within the network, e.g., that all triads have the same probability of closure, the relational theoretical processes, which are central to many political processes, are localized to particular parts of the network.

Coalition-building (e.g., international alliance coalitions, voting coalitions in legislatures, building voter support for a political candidate), for example, is often theorized as an outcome of two processes—balancing, or formation of blocs in other parts of ideological spectrum (Morrow 1991; Kedar 2005)—and bandwagoning, or the tendency of weaker players to align with the expected winner (Sweeney and Fritz 2004; Hassell 2016). In the traditional network-theory conceptualization of an alliance of two states or two legislatures as an edge, the balancing process may predict that the realization of any given edge is conditional on realization of edges located in the opposite part of the ideological spectrum: i.e. the competing coalitions form in response to one another. The bandwagoning process, in contrast, may predict that edge formation will cluster within ideological space: i.e. formation of a voting alliance will likely trigger additional allies to jump on-board.

While one may, of course, try to imagine possible ways to model coalition formation within the ERGMs set-up (e.g., using triads or two-stars), any Markovian features would get at the theorized processes only indirectly. A coefficient on triads, for example, would capture the average tendency of any two actors within the network to form an alliance, given that they are both allied with the same third actor. Such an approach, however, would not allow for localizing alliance formation to particular parts of the network (e.g., ideationally similar states are more likely to ally).

LSMs, in turn, allow for a hierarchical approach to modeling network data, in which dependence is accounted for by considering the relevant node-specific latent variables that are sources of non-independence, such as group membership or position within social space (Hoff and Ward 2004; Minhas, Hoff, and Ward 2016). Just like with ERGMs, LSMs focus on sources of dependence associated with nodes, whereas many important applications require modeling dependence among edges.

The third type of network modeling, or, to be more precise, "spatial" modeling—CARs and SARs—accounts for non-independence among observations by including lag structures which measure theorized sources of connectivity (Anselin 2013; Besag 1974; Hays, Kachi, and Franzese 2010). While existing social science applications of spatial autoregression are also limited to modeling diffusion among nodes, rather than edges, we propose an extension that would allow for precisely that.

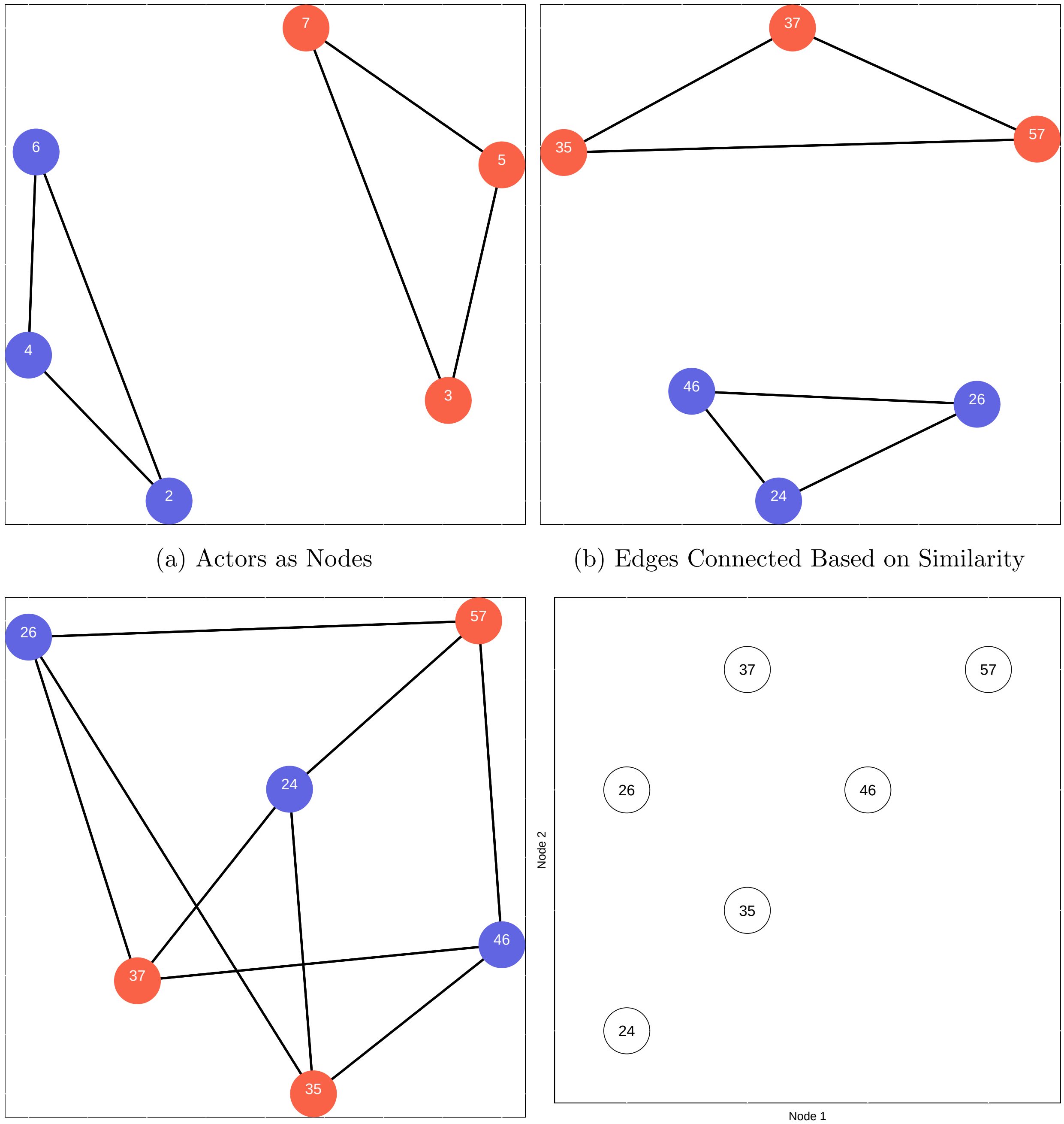
2.1 Conceptualizing Connectivities among Network Edges

To illustrate the theory behind our approach, let us start with a hypothetical example of a network that consists of a number of nodes, some of which are connected by an edge, as shown in Subfigure 1a of Figure 1. In most of the traditional network research, nodes in this network would represent individual actors, such as legislators, and edges would correspond to some type of relationship among nodes, such as an action of cosponsoring the same piece of legislation. Any two legislators then would then share an edge if they both cosponsored the same piece of legislation.

This traditional framework is useful for studying individual-level factors that may lead legislators to cosponsor legislation, such as party identification, seniority, home-state population, etc., or dyadic-level factors, such as whether pairs of legislators are from the same party or similar in seniority. Formation of legislative coalitions in support or opposition to legislation, however, is not easily modeled using either individual or dyadic covariates. Formation of legislative coalitions is a *localized network process*, in which network edges realize in response to other edges. In a two-party system, for example, each pair of legislators may cosponsor a bill to either signal their support for the bill to members of their own party or to balance a bill that is gaining cosponsors among the opposing party.

Modeling such localized processes is facilitated by adopting more flexible assumptions on

Figure 1: Alternative Conceptualizations of Nodes and Edges



(a) Actors as Nodes

(b) Edges Connected Based on Similarity

(c) Edges Connected Based on Dissimilarity

(d) Edges Connected Based on Distance

what constitutes network nodes and edges. Modeling the formation of legislative cosponsorships, for instance, is made more straightforward if we treat cosponsorships as nodes connected to one another based on the ideological similarity among the legislators that form them. This re-conceptualization is shown in Subfigure 1b, in which each edge from Subfigure 1a is now depicted as a node, and relationships between each pair of edges (whether they connect two nodes of the same color) as edges. For example, node 24 in Subfigure 1b corresponds to the edge between nodes 2 and 4 in Subfigure 1a, and nodes 35 and 57 are connected by an edge because they both connect nodes of the same color in Subfigure 1a. Returning to the cosponsorships example, this framework allows us to model cosponsorships as a tipping-point or bandwagoning process: initial introduction triggers a cascade of cosponsorships among ideologically similar legislators. Alternatively, growing support for a bill may lead to a balancing process, i.e., an introduction of an opposing bill by the ideologically dissimilar legislators. Then the network of cosponsorships would look like that depicted in Subfigure 1c, in which two cosponsorshios are connected if they connect nodes of different colors. Second-degree edges may also be continuous, i.e. in Subfigure 1a edges are placed in a two-dimensional space with x- and y- coordinates corresponding to the numerical identifiers of the nodes that make up each edge. In the latter example, the strength of connections among edges is measured using the Euclidean distance, i.e. edges 24 and 35 are separated by a shorter Euclidean distance and, therefore, have a stronger connection than edges 24 and 57.

3 Statistical Estimation of Local Structures within Networks

In this section, we demonstrate a statistical approach to modeling localized network outcomes using the framework of the Markov random field models (Casleton, Nordman, and Kaiser 2016; Kaiser and Caragea 2009). Suppose i is a potential edge in a network of n edges (e.g., a cosponsorship in the example discussed above), $i = \{1, 2, \ldots, n\}$, so that i's location is denoted as $s_i = (u_i, v_i)$ in Cartesian space. Next, define i's neighbors as N_i, so that $\boldsymbol{y}(N_i)$

is a vector of outcomes in i's neighbors and $\boldsymbol{y}(N_i)\{\boldsymbol{y}(s_j) : s_j \neq s_i\}$. If dependencies among edges are binary, then the next step if to make a Markov assumption of conditional spatial independence of the form:

$$f(y(s_i)|\mathbf{y}(s_j) : s_j \neq s_i) = f(y(s_i)|\mathbf{y}(N_i)) \tag{1}$$

Thus, in the case of binary dependencies among edges, the realization of any given edge i is dependent on realization of every other edge in its neighborhood N_i, yet conditionally independent of realization of edges in its neighbors' neighborhoods. Intuitively, this assumption simply means that i is affected by its immediate neighbors rather than by its neighbors' neighborhoods.

Note that, if dependencies among edges are measures on a continuous scale, as is the case of interest here, we can simply define i^s neighbors as $-i$, so that $\boldsymbol{y}_{-i} = \boldsymbol{y}(s_{-i}) = \{\boldsymbol{y}(s_j) : s_j \neq s_i\}$. In case of continuous dependencies, the Markov assumption (1) is redundant.

Further, denote the binary random variable, $y(s_i) = y_i$, that records the presence or absence of an edge, such that:

$$y(s_i) = \begin{cases} 1 & \text{if edge} \quad \text{is present} \\ 0 & \text{if edge} \quad \text{is absent.} \end{cases}$$

Two edges are assumed to be connected by an edge if they belong to the same neighborhood(s), with neighborhoods defined based on the theorized process. More formally, we can say that an edge i is conditionally independent of edge j unless j is a neighbor or i (and hence j is a neighbor of i). Of course, if second degree edges are measured on a continuous scale, then the realization of edge i is dependent on realizations of all other edges (every edge is in every other edge's neighborhood). Each neighborhood is measured via an n-by-n matrix $\mathbf{W}$, whose ij cell is a binary or a continuous measure of connectivity between edges

i and j and with 0s on the major diagonal (edges have no connectivity with themselves). In political-science applications, the connectivity matrix $\mathbf{W}$ may represent physical or geographical distance between edges, their ideological similarity, or any other pairwise measures of relationship.

Consistent with the legislative example above, we assume a binary conditional distribution, which is expressed in exponential family form as:

$$\mathrm{P}(Y_i = y_i|\boldsymbol{y}(N_i)) = \exp\left[A_i(\boldsymbol{y}(N_i))y_i - B(\boldsymbol{y}(N_i))\right], \tag{2}$$

where A_i is a natural parameter function and $B_i = \log[1 + A_i\left(\mathbf{y}(N_i)\right)]$. Conditional dependencies among edges are modeled through the natural parameter function as:

$$A_i(\mathbf{y}(N_i)) = \log\left(\frac{\kappa_i}{1-\kappa_i}\right) + \eta \sum_{j \in N_i} w_{ij}(y_j - \kappa_j), \tag{3}$$

where $\log\left(\frac{\kappa_i}{1-\kappa_i}\right) = X_i^T\boldsymbol{\beta}$, X_i is a vector of exogenous covariates, $\boldsymbol{\beta}$ is a vector of estimation parameters, w_{ij} is the ij^{th} element of a matrix of connectivities among edges, $\mathbf{W}$, η is a dependence parameter, and y_j is the outcome in location s_j. Parameters $\boldsymbol{\beta}$ are associated with the instantaneous effects of the exogenous covariates, while the dependence parameter η represents the dependence among observations, with positive values indicating a direct relationship between edge realizations in neighboring units and negative values indicating an inverse relationship.

The formulation of the spatial dependence term, $\eta\sum_{j=1}^{n} w_{ij}(y_j - \kappa_j)$, ensures that it can make a positive or a negative contribution to the natural parameter function. This term increases the value of the natural parameter function if the realization of the neighbors' values exceeds its expectation, $y_j > \kappa_j$, and decreases its value if the observed value is less than the expected value, $y_j < \kappa_j$. Since in the binary case $y_j \in \{0, 1\}$ and $0 < \kappa_j < 1$, a positive dependence parameter, $\eta > 0$, indicates that an absence of edges in neighboring locations, $y_j = 0$, has a negative effect on the probability that $y_i = 1$, and the presence of edges in neighboring locations, $y_j = 1$, has a positive effect. Analogously, a negative dependence parameter, $\eta < 0$, implies the opposite: the absence of edges in neighboring locations, $y_j = 0$, has a positive effect on the probability of edge realizations in $y_i = 1$, and the presence of edges in neighboring locations, $y_j = 1$, has a negative effect.

If the connectivity among nodes, $\mathbf{W}$, is measured on a continuous scale, s.t. larger values of w_{ij} denote larger differences between i and j, then a positive dependence parameter, $\eta > 0$, indicates that a presence of an edge in a distant location $y_j = 1$ has a positive effect on the probability of edge realizations in y_i, which is consistent with balancing. A negative dependence parameter, $\eta < 0$, in contrast, would indicate that a presence of an edge in a distant location has a negative effect on the probability of $y_i = 1$, which is consistent with such processes as clustering.

As is the case for the general class of Markov random field models, of which the model above is a special case, the specification of full conditional distribution leads to a valid joint distribution under certain conditions (Kaiser and Cressie 2000). For the LSGM in Equation 2, one of these conditions is that the connectivity matrix $\mathbf{W}$ be symmetric for all pairs of edges, i.e. $w_{ij} = w_{ji}$. This symmetry condition, of course, implies that, in contrast to the standard specification of SAR models, the connectivity matrix must not be row standardized, as row-standardizing will violate this assumption.[1] Model parameters may be obtained by maximizing a log pseudo-likelihood (PL), which is a summation of the log of the conditional distributions (Besag 1975):

$$\log PL = \sum_i \{y_i \log(p_i) + (1 - y_i) \log(1 - p_i))\}, \tag{4}$$

[1] One benefit of this is that no standardization assumption is necessary.

where

$$p_i = \frac{\exp(A_i(\mathbf{y}(N_i)))}{1 + \exp(A_i(\mathbf{y}(N_i)))} \tag{5}$$

The point estimates recovered by maximizing the PL function have been shown to be consistent for the general case of Markov random fields models (Casleton, Nordman, and Kaiser 2016; Guyon 1995). Standard errors may be obtained via parametric bootstrap or, under certain assumptions, calculated via a Godambe information matrix (Godambe and Kale 1991). In what follows, we use Monte Carlo simulations to demonstrate the properties of the parameter estimates for the special case of the model presented in Equation 2, and follow up with two empirical application to data on international alliances and legislative cosponsorships.

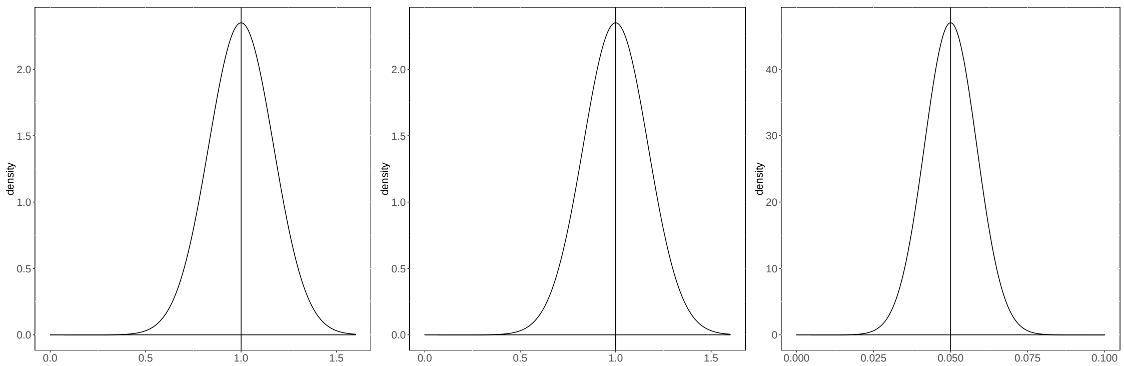
Unlike the SAR model, LSGM is easily generalized to other distributions within the exponential family by simply re-specifying the natural parameter function (Casleton, Nordman, and Kaiser 2016). Thus, the above example could be re-formulated to model continuous, multinomial, ordered, or count data. In this respect, LSGM, and Markov random fields models more broadly,[2] also present a more general modeling approach that does not require assumptions related to latent variable distributions, as is the case with SAR models commonly used in political science.

4 Monte Carlo Simulations

We start by generating information for 100 observations (nodes) with characteristics captured by variable X_i, drawn from a standard normal distribution. We proceed to convert these data to a dyadic format (edges), $i = 1, 2, \ldots, n$ by pairing each observation with each other observation and omitting self-referencing pairs of the type $i - i$ for a total of $n = 9900$ edges.

[2]CAR models are, of course, a type of MRF models.

Figure 2: Monte Carlo Results for Parameter Estimates



(a) β_0 (b) β_1 (c) η

Note: Vertical lines represent the true values of the parameters. The curves represent kernel density graphs of the estimates within 90% confidence intervals.

To generate a meaningful connectivity matrix, $\mathbf{W}$, we place each pair on an evenly spaced ten-by-ten grid and calculate the Euclidean distance between the two units making up each edge. Next, we use a Gibbs sampler with randomly initialized values to generate random variable, $Y(s_i)$.

The Gibbs sampler starts with the randomly generated starting values for the dependent variable and iteratively updates them, one observation at a time, using the specified parameter values, $\beta_0 = \beta_1 = 1$ and $\eta = .05$, following the data-generating process specified in Equation 2. The Gibbs sampler was run with a burn-in of 20,000, after which sample graphs were retained from 100,000 subsequent rounds with 50 iterations for thinning.

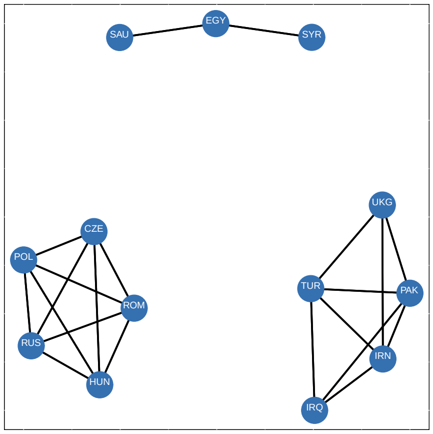
The results of the Monte Carlo simulations are presented in Figure 2. As expected, the 90% confidence intervals, displayed in the figure, converge around the true value of each parameter. The positive coefficient η indicates the presence of a direct dependence among the realizations of neighboring edges.

5 Application: Formation of International Alliances

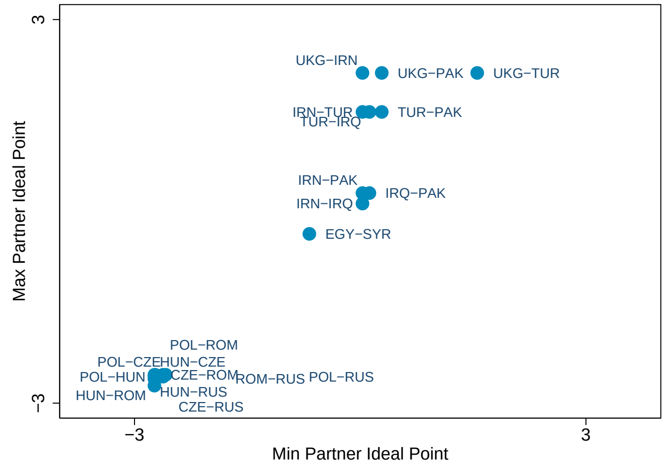
To further demonstrate the benefits of LSGM, we apply it to modeling the formation of the international alliance network. One prominent theory in alliance research suggests that alliances tend to form among states with similar policy orientations (Gibler and Rider 2004; Lai and Reiter 2000). The logic is that ideationally similar states will naturally prefer to join their forces to counter a common threat. This reasoning leads to several empirical expectations. If we think of international alliances as network nodes and ideational distances among them as network relationships, then the first expectation is that we should observe that alliances will not be uniformly distributed within the ideational space, but will cluster in opposite parts of of it. Moreover, there may be a balancing process, in which formation of an alliance in one part of the ideational space will trigger a balancing act in the opposite part of the ideational space (Schweller 2004). As most alliances are multilateral (Gibler and Wolford 2006), we may also expect clustering of alliances within ideational space.

Figure 3 provides a visual demonstration of the modeling approach, and how it differs from a more traditional network theories of alliances. Subfigure 3a shows a traditional visualization of the alliance network with countries as nodes that are connected by edges if the given pair of states were part of an alliance in a given year. Such a visualization corresponds to the traditional theoretical framework within the alliance research—a framework that models alliance formation as a function of state-level (node-level) and dyadic (edge-level) covariates, such as joint democracy, military power or asymmetry, and bilateral trade (Lai and Reiter 2000). Many important theories of alliance formation, such as "birds of a feather," as well as balancing and bandwagoning, posit processes that cannot be directly modeled using state- and dyad-level attributes—processes that involve alliances (edges) forming in response to other alliances that formed within the network. Modeling these important theoretical processes requires reconceptualizing the alliance network as a network in which alliances

Figure 3: Visualizing International Alliance Formation, 1955



(a) Alliance Connections among States



(b) Alliances within Ideational Space

Notes: Alliance data are obtained from the Correlates of War Project (Gibler 2009).

themselves are treated as nodes, and relationships among them are treated as edges.

Subfigure 3b demonstartes such a reconceptualization. Each bilateral alliance relationship is represented as a node, that is placed in an ideational Cartesian space in accordance with the the ideational scores of each of the two allies that serve as the x and y coordinates.[3] The Euclidean distance between each pair of alliances then serves as a proxy for the ideational dissimilarity between them (a continuous conceptualization of a relationship/edge).

This visualization of the international alliance network mimics the theoretical processes posited by the the "birds of a feather" theory of alliance formation. Conceptualizing of alliances in these terms uncovers a number of dynamics, consistent with this theoretical framework. For example, Subfigure 3b shows that that international alliances tend to form between ideologically similar rather than different states—most alliances cluster close to the

[3]We utilized ideal point scores, developed by Bailey, Strezhnev, and Voeten (2015). Bailey, Strezhnev, and Voeten (2015) use UN voting data to align all countries on a standardized normal scale (from about -3 to +3) between 1947-2012, where higher scores are associated with the US and its allies and lower scores are attributed to Russia/Soviet Union block. In Figure 3b, each alliance partner ideal point score provides one of the alliance coordinates in a two dimensional space, and ideological distance is calculated as the Euclidean distance between alliances.

diagonal of the graph (the line $y = x$ would represent the location of all alliance partners with identical ideal scores) rather than in the areas off the diagonal. While this pattern is expected, it is nonetheless useful to be able to confirm this intuition by visualizing the data in a relevant way. Second, Subfigure 3b highlights clustering in two opposite areas of the ideological space, which is consistent with the balancing logic described above. Third, Subfigure 3b reveals some insights regarding the ideational cohesion within each of the opposing blocs that formed in the given year: the Soviet bloc consisting of alliances among Russia, Czechoslovakia, Hungary, Poland, and Romania is much more concentrated within the ideational space that the bloc among the United Kingdom, Turkey, Pakistan, Iran, and Iraq.

Of course, possible connectivities among alliance edges are not limited to proximity/distance of alliance-edges within ideational space. Alternative sources/conceptualization of connectivity among alliances may focus on whether pairs of alliances connect similar states (e.g., two democracies) or share a common node. One may align alliance-edges in different types of two-dimensional space, e.g. using various state-level attributes as coordinates.

In order to perform a statistical test of the balancing and clustering hypotheses described above, we use international alliance formation data from the Correlates of War Project (Gibler 2009). The dependent variable is a dichotomous measure of whether a pair of states were part of an alliance in a given year. The estimation sample consists of all politically relevant pairs of states between 1946-2000; the unit of analysis is a network edge (formation/presence of an alliance).

A metric to define connectivity $\mathbf{W}$ between alliance is measured using each partner's ideal scores based on United Nations General Assembly voting (Bailey, Strezhnev, and Voeten 2015). We treat each potential ally's ideal score as a coordinate, which allows us to align all potential alliances in a two-dimensional space. Each ij cell of the $\mathbf{W}$ matrix thus contained a measure of the Euclidean distance between i and j in this two-dimensional ideological space. Shorter distances indicate policy similarity while greater distances indicate policy

Table 1: Applying LSGM to Model International Alliance Formation, 1946-2000

Edge Connectivity:		
Ideational Distance	0.016*	(0.001)
Military Power Ratio	−2.363*	(0.073)
Dyadic Trade	0.015*	(0.005)
Joint Democracy	0.884*	(0.024)
Constant	0.094	(0.072)

Note: $^{*}p < 0.05$ Standard errors are obtained using a parametric bootstrap (1100 simulations of complete networks, 100 burnin and 50 iterations for thinning).

dissimilarity.

Finally, we include several control variables measured at the state-dyad-level. Consistent with prior research, we expect that pairs of states are more likely to be part of a military alliance if they engage in international trade and are jointly democratic (Lai and Reiter 2000). We also expect that states are more likely to ally if they are approximately even in terms of military capabilities (Kimball 2006). Data on international trade are obtained from the Correlates of War Project (Barbieri, Keshk, and Pollins 2009), and data on levels of democracy are obtained from Marshall and Jaggers (2014). *Military Power Ratio* is measured as the ratio of the military capabilities of the more powerful state in a pair of states to the total military capabilities of the pair, or $\frac{max(m_1,m_2)}{m_1+m_2}$. Data on military symmetry/asymmetry are obtained from Arena (2016).

The results of the estimation are presented in Table 1. The coefficient on *Ideational Distance* is positive, indicating a balancing process: alliances-edges form in response to other alliances-edges that realize in a ideationally different part of the network. This finding is consistent with the balancing logic above, in which ideationally similar states balance against the growing power of their adversaries. This resonates with a neoclassical version of the realist balancing theory that qualifies the neorealist balancing hypothesis by highlighting domestic preferences.

The coefficients on the control variables are as expected. *Military Power Ratio* has a negative effect, suggesting that symmetric alliances are more common than asymmetric ones. *Dyadic Trade* and *Joint Democracy* have a positive effect, indicating that trade and similar political institutions enhance military cooperation.

6 Application: Formation of Legislative Coalitions

In this section, we demonstrate an empirical application of LSGM to modeling legislative cosponsorships in the Senate of the 107th Congress (2001-2003). We treat each pair of senators as a network edge, which is realized (takes on the value of 1) if two senators cosponsored a piece of legislation; if the pair are not part of a joint cosponsorship, the edge between them is coded as 0. We posit that legislative cosponsorships-edges are most likely to form in response to other cosponsorship within the same issue area: thus, legislators from the opposite parties may cosponsor competing pieces of legislation related to the same issue. For example, ideologically liberal senators may cosponsor a bill stipulating an increase in minimum wage in response to a piece of ideologically conservative legislature aimed at relaxing wage standards. Likewise, once a bill on a given issue is introduced, legislators of similar political ideology are likely to form cosponsorships with the original sponsor and each other. To zero in on the process of such counter-balancing within an issue area, we narrow our focus to the bills that are broadly related to labor, employment, and pensions, as well as the relevant appropriation decisions, as coded by (Adler and Wilkerson 2006). Data on cosponsorships were obtained from (Fowler 2006*a*,*b*). Analogous to the alliance example above, cosposorship-edges are treated as located within an ideational space; each cosponsorship is mapped in a Cartesian space using the DWNominate scores of the corresponding pair of senators as coordinates (Poole and Rosenthal 2011).

Table 2 presents the results of the estimation. The coefficient on the *Ideological Distance* is negative and statistically significant: cosponsorships cluster within ideational space. This

Table 2: Applying LSGM to Model Senate Cosponsorships

Edge Connectivity:		
Ideological Distance	-1.235^{*}	(0.519)
Same Party	0.704^{*}	(0.051)
Labor Committee	0.149^{*}	(0.044)
Minimum Seniority	-0.047^{*}	(0.010)
Constant	0.387^{*}	(0.089)

Notes: Standard errors were obtained using a parametric bootstrap via a Gibbs sampler of 300 complete simulations (50 for burnin and thinning.)

indicates that cosponsorship behavior is more likely, on average, to happen as a result of bandwagoning than balancing. Most of the control variables act in expected directions. The coefficient on *Same Party* is positive and statistically significant, suggesting that consponsorships are more likely among members of the same party. The coefficient on *Labor Committee* is positive and statistically significant, consistent with the logic that cosponsorships on labor legislation are more likely to happen among a pair of legislators if at least one member of the pair is part of the Health, Education, Labor, and Pensions Senate Committee. The coefficient on *Minimum Seniority* is negative and statistically significant, which indicates that senior pairs of legislators are less likely to cosponsor legislation than pairs with at least one junior legislator.

7 Conclusion

This paper introduces an LSGM—a statistical estimator designed for modeling the formation of local structures within networks. We demonstrated the desirable asymptotic properties of the estimator using Monte Carlo simulations and provided two illustrative applications to modeling the formation of the international alliance network and legislative coalitions. More broadly, we emphasized the narrowness and inflexibility of the traditional network focus on actors as nodes and relationships among them as edges. Adopting more flexible assumptions of what constitutes nodes and edges helps model many localized network processes, such as balancing, bandwagoning, and cascades.

LSGM has many potential applications to modeling information diffusion, or tipping-point processes, such as community outreach related to building support for a particular policy. The proposed framework easily extends to modeling localized formation of other types of network structures, such as triangles or k-stars, albeit the theoretical mechanisms behind such processes are currently under-developed. The LSGM provides a tool for testing for such dependencies in a controlled, interpretable way.

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